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# On the Connection of Leptogenesis with Low Energy $CP$ Violation and LFV Charged Lepton Decays

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## Abstract

Assuming only a hierarchical structure of the heavy Majorana neutrino masses and of the neutrino Dirac mass matrix  $m_D$  of the see-saw mechanism, we find that in order to produce the observed baryon asymmetry of the Universe via leptogenesis, the scale of  $m_D$  should be given by the up-quark masses. Lepton flavor violating decays  $\mu \rightarrow e + \gamma$ ,  $\tau \rightarrow \mu + \gamma$  and  $\tau \rightarrow e + \gamma$  are considered and a characteristic relation between their decay rates is predicted. The effective Majorana mass in neutrinoless double beta decay depends on the  $CP$  violating phase controlling the leptogenesis if one of the heavy Majorana neutrinos is much heavier than the other two. Successful leptogenesis requires a rather mild mass hierarchy between the latter. The indicated hierarchical relations are also compatible with the low-energy neutrino mixing phenomenology. The scenario under study is compatible with the low-energy neutrino mixing phenomenology. The  $CP$  violation effects in neutrino oscillations can be observable. In general, there is no direct connection between the latter and the  $CP$  violation in leptogenesis. If the  $CP$  violating phases of the see-saw model satisfy certain relations, the baryon asymmetry of the Universe and the rephasing invariant  $J_{CP}$  which determines the magnitude of the  $CP$  violation effects in neutrino oscillations, depend on the same  $CP$  violating phase and their signs are correlated.

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# 1 Introduction

Explaining both the observed baryon asymmetry of the Universe ( $Y_B$ ) and the smallness of the neutrino masses, can be done successfully by combining the see-saw [1] and the leptogenesis [2] mechanisms. The first predicts the neutrinos to be Majorana particles and introduces additional right-handed heavy Majorana neutrinos, whose out-of-equilibrium decay in the early Universe generates  $Y_B$  via the second. The mounting evidence in favor of neutrino oscillations (see, e.g., [3, 4, 5, 6]) gave analyzes of these two mechanisms a firmer phenomenological basis. Establishing a connection between the low energy (neutrino mixing) and high energy (leptogenesis) parameters has gathered much attention in recent years [7, 8, 9, 10, 11, 12, 13, 14, 15]. However, the number of parameters of the see-saw mechanism is significantly larger than the number of quantities measurable in the “low energy” neutrino experiments. It is the neutrino Dirac mass matrix  $m_D$  that contains most of the unknown parameters and on whose knowledge any statement about a possible connection of low and high energy phenomena relies. While, in general, it is impossible to establish a direct link between the phenomena related to neutrino mixing and to leptogenesis, most specific models usually allow very well for such a connection.

In the present article we investigate the possible link between the leptogenesis, taking place at “high energy”, and the “low energy” phenomena associated with the existence of neutrino mixing. Our analysis is based on the assumption of a hierarchical structure of the heavy Majorana neutrino masses and of the Dirac mass matrix  $m_D$ , both of which are part of the see-saw mechanism. As specific examples of low energy phenomena related to neutrino mixing we consider the lepton flavour violating (LFV) charged lepton decays,  $\mu \rightarrow e + \gamma$ ,  $\tau \rightarrow \mu + \gamma$  and  $\tau \rightarrow e + \gamma$ , the effective Majorana mass in neutrinoless double beta  $((\beta\beta)_{0\nu})$  decay,  $|\langle m \rangle|$ , and the  $CP$  violating asymmetry in neutrino oscillations. We analyze, in particular, the effects of the high energy  $CP$  violating phases, which control the generation of the baryon asymmetry in the leptogenesis scenario, in  $|\langle m \rangle|$  and in the leptonic  $CP$  violation rephasing invariant  $J_{CP}$  [16] which determines the magnitude of the  $CP$  violation in neutrino oscillations <sup>1</sup>.

In the next Section we briefly review the formalisms associated with the see-saw mechanism and with the leptogenesis scenario, and that of LFV charged lepton decays. Different parameterizations of  $m_D$  used in the literature and the corresponding connection they imply between the high and low energy physical parameters are discussed in Section 3. In Section 4 we formulate the assumptions of a hierarchical structure of the heavy Majorana neutrino masses and of the Dirac mass matrix  $m_D$  on which our investigation is based, and analyze the leptogenesis in the so-called “bi-unitary” parametrization of  $m_D$ . Predictions for the branching ratios of the LFV charged lepton decays are also given. In Section 5 the effects of the high energy leptogenesis  $CP$  violating phases on  $|\langle m \rangle|$  and on the leptonic  $CP$  violation rephasing invariant  $J_{CP}$ . Finally, we conclude in Section 6.

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<sup>1</sup>For similar earlier attempts (not taking into account the questions of  $CP$  violation and LFV decays), see, e.g., [17, 18].

## 2 Framework

The neutrino oscillation data can consistently be described within a 3-neutrino mixing scheme with massive Majorana neutrinos, in which the light neutrino Majorana mass matrix  $m_\nu$  is given by:

$$m_\nu = U_{\text{PMNS}} m_\nu^{\text{diag}} U_{\text{PMNS}}^T . \quad (1)$$

Here  $m_\nu^{\text{diag}}$  is a diagonal matrix containing the masses  $m_{1,2,3}$  of the three massive Majorana neutrinos and  $U_{\text{PMNS}}$  is the unitary Pontecorvo–Maki–Nagakawa–Sakata [19] lepton mixing matrix, which can be parametrized as

$$U_{\text{PMNS}} = V \text{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)}) \\ = \begin{pmatrix} c_1 c_3 & s_1 c_3 & s_3 e^{-i\delta} \\ -s_1 c_2 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 - s_1 s_2 s_3 e^{i\delta} & s_2 c_3 \\ s_1 s_2 - c_1 c_2 s_3 e^{i\delta} & -c_1 s_2 - s_1 c_2 s_3 e^{i\delta} & c_2 c_3 \end{pmatrix} \text{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)}) , \quad (2)$$

where  $\delta$  is a Dirac  $CP$  violating phase,  $\alpha$  and  $\beta$  are Majorana  $CP$  violating phases [20, 21],  $c_i = \cos \theta_i$  and  $s_i = \sin \theta_i$ . The angles  $\theta_1$  and  $\theta_2$  control the oscillations of solar and atmospheric neutrinos, respectively. The angle  $\theta_3$  is limited by the CHOOZ and Palo Verde reactor  $\bar{\nu}_e$  experiments: one has  $\sin^2 \theta_3 < 0.05$  [22, 5]. The Dirac phase  $\delta$  can be measured, in principle, in long base-line neutrino oscillation experiments (see, e.g., [23, 24]). The flavour neutrino oscillations are insensitive to the Majorana phases  $\alpha$  and  $\beta$  [20, 25]. Information about these phases can be obtained by studying processes in which the total lepton charge  $L$  is not conserved and changes by two units [26, 27, 28, 29, 30]:  $(\beta\beta)_{0\nu}$ -decay,  $K^+ \rightarrow \pi^- + \mu^+ + \mu^+$ , etc.

Introducing a Dirac neutrino mass term and a Majorana mass term for the right-handed neutrinos via the Lagrangian

$$-\mathcal{L} = \overline{\nu_{Li}} (m_D)_{ij} N_{Rj} + \frac{1}{2} \overline{(N_{Ri})^c} (M_R)_{ij} N_{Rj} , \quad (3)$$

leads for sufficiently large  $M_R$  to the well know see-saw [1] formula

$$m_\nu \simeq -m_D M_R^{-1} m_D^T , \quad (4)$$

where terms of order  $\mathcal{O}(M_R^{-2})$  are neglected. In order to explain the smallness of neutrino masses, the see-saw mechanism requires the existence of heavy right-handed Majorana neutrinos. The latter can create a lepton asymmetry via their  $CP$  violating out-of-equilibrium decays induced by the see-saw related Yukawa couplings. At later epoch the lepton asymmetry is converted into the baryon asymmetry of the Universe through sphaleron mediated processes [2]. Thus, leptogenesis is naturally incorporated in the see-saw model, which makes the model particularly attractive.

### 2.1 Leptogenesis

Leptogenesis fulfills all of Sakharov's three conditions [31] for generation of a non-vanishing baryon asymmetry  $Y_B$ . The requisite  $CP$  violating asymmetry is caused by the interference

of the tree level contribution and the one-loop corrections in the decay rate of the lightest of the three heavy Majorana neutrinos,  $N_1 \rightarrow \Phi^- \ell^+$  and  $N_1 \rightarrow \Phi^+ \ell^-$ :

$$\begin{aligned} \varepsilon_1 &= \frac{\Gamma(N_1 \rightarrow \Phi^- \ell^+) - \Gamma(N_1 \rightarrow \Phi^+ \ell^-)}{\Gamma(N_1 \rightarrow \Phi^- \ell^+) + \Gamma(N_1 \rightarrow \Phi^+ \ell^-)} \\ &\simeq \frac{1}{8\pi v^2} \frac{1}{(m_D^\dagger m_D)_{11}} \sum_{j=2,3} \text{Im}(m_D^\dagger m_D)_{1j}^2 (f(M_j^2/M_1^2) + g(M_j^2/M_1^2)) . \end{aligned} \quad (5)$$

Here  $v \simeq 174$  GeV is the electroweak symmetry breaking scale. The function  $f$  stems from vertex [2, 32] and  $g$  from self-energy [33] contributions:

$$f(x) = \sqrt{x} \left( 1 - (1+x) \ln \left( \frac{1+x}{x} \right) \right) , \quad g(x) = \frac{\sqrt{x}}{1-x} . \quad (6)$$

For  $x \gg 1$ , i.e., for hierarchical heavy Majorana neutrinos, one has  $f(x) + g(x) \simeq -\frac{3}{2\sqrt{x}}$ . The baryon asymmetry is obtained via

$$Y_B = a \frac{\kappa}{g^*} \varepsilon_1 , \quad (7)$$

where  $a \simeq -1/2$  is the fraction of the lepton asymmetry converted into a baryon asymmetry [34],  $g^* \simeq 100$  is the number of massless degrees of freedom at the time of the decay, and  $\kappa$  is a dilution factor that is obtained by solving the Boltzmann equations. Typically, one gets  $Y_B \sim 10^{-10}$  when  $\varepsilon_1 \sim (10^{-6} - 10^{-7})$  and  $\kappa \sim (10^{-3} - 10^{-2})$ . We note that this estimate of  $Y_B$  is valid in the supersymmetric (SUSY) theories as well since in the latter both  $g^*$  and  $\varepsilon_1$  are larger approximately by the same factor of 2.

## 2.2 LFV Charged Lepton Decays

In extensions of the Standard Theory including massive neutrinos, lepton flavour non-conserving (LFV) processes such as  $\mu \rightarrow e + \gamma$ ,  $\mu \rightarrow 3e$ ,  $\tau \rightarrow \mu + \gamma$ ,  $\tau \rightarrow e + \gamma$ , etc. are predicted to take place (see, e.g., [35]). However, in the non-SUSY versions of the see-saw model, the decay rates and cross sections of the LFV processes are strongly suppressed and are practically unobservable [36, 37]. The most stringent experimental limit on the  $\mu \rightarrow e + \gamma$  decay branching ratio reads [38]

$$BR(\mu \rightarrow e + \gamma) < 1.2 \times 10^{-11} . \quad (8)$$

There are prospects to improve this limit by  $\sim 3$  orders of magnitude in the future (see, e.g., [39]). The existing bounds on the LFV decays of the  $\tau$ -lepton are considerably less stringent: one has, e.g.,  $BR(\tau \rightarrow \mu + \gamma) < 6.0 \times 10^{-7}$  [40]. There are possibilities to reach a sensitivity to values of  $BR(\tau \rightarrow \mu + \gamma) \sim (10^{-8} - 10^{-9})$  at B-factory and LHC experiments [41].

In SUSY theories incorporating the see-saw mechanism, new sources of lepton flavour violation arise. In SUSY GUT theories these are typically related to the soft SUSY-breaking terms of the Lagrangian. Under the assumption of flavour universality of the SUSY breaking sector (scalar masses and trilinear couplings) at the GUT scale  $M_X$ , new lepton flavour non-conserving couplings are induced at low energy by the matrix of neutrino Yukawa couplings

$m_D/v$  through renormalization group running of the scalar lepton masses. The latter generates non-zero flavour non-diagonal entries in the slepton mass matrices and in the trilinear scalar couplings. These were shown to be proportional to  $m_D m_D^\dagger$  [42]. More specifically, using the leading-log approximation, one can estimate [42, 43, 44] the branching ratio of the charged lepton decays  $\ell_i \rightarrow \ell_j + \gamma$ ,  $\ell_i(\ell_j) = \tau, \mu, e$  for  $i(j) = 3, 2, 1$ ,  $i > j$ ,

$$BR(\ell_i \rightarrow \ell_j + \gamma) \simeq \alpha^3 \left( \frac{(3 + a_0) m_0^2}{8 \pi^2 m_S^4 G_F v^2} \right)^2 \left| (m_D L m_D^\dagger)_{ij} \right|^2 \tan^2 \beta, \quad (9)$$

where  $m_S$  denotes a slepton mass,  $m_0$  is the universal mass scale of the sparticles and  $a_0$  is a trilinear coupling (all at  $M_X$ ). The diagonal matrix  $L$  reads:  $L = \text{diag}(L_1, L_2, L_3)$ , where  $L_i = \log M_X/M_i$ . To get a feeling for the numbers involved, for  $a_0 = \mathcal{O}(1)$ ,  $m_S \sim m_0 \sim 10^2$  GeV,  $M_X \sim 10^{16}$  GeV and  $M_i \sim 10^{10}$  GeV, one finds

$$BR(\ell_i \rightarrow \ell_j + \gamma) \sim 10^{-15} \tan^2 \beta \left| \frac{(m_D m_D^\dagger)_{ij}}{\text{GeV}^2} \right|^2. \quad (10)$$

This is within the planned sensitivity of the next generation experiments. The predicted values of  $BR(\ell_i \rightarrow \ell_j + \gamma)$  are strongly dependent on the SUSY parameters, a topic beyond the scope of the present article.

Since the rates of LFV charged lepton decays depend on  $m_D m_D^\dagger$ , the decay asymmetry  $\varepsilon_1$  depends on  $m_D^\dagger m_D$  and leptonic  $CP$  violating effects originate from  $m_\nu \sim m_D m_D^T$ , one might in a given model expect some interplay between these three phenomena. We shall investigate this in Sections 4 and 5.

## 3 How it Works

### 3.1 Parameterizations

At low energy and in the case of  $N$  flavour neutrinos, we can measure, in principle, the  $N \times N$  light neutrino mass matrix  $m_\nu$ . It contains  $N$  mass eigenstates,  $\frac{1}{2}N(N-1)$  mixing angles and  $\frac{1}{2}N(N-1)$  phases, thus altogether  $N^2$  measurable quantities. Of these  $\frac{1}{2}N(N+1)$  are real and  $\frac{1}{2}N(N-1)$  are  $CP$  violating phases. One can easily count the number of *all see-saw* parameters in the weak basis, in which both,  $M_R$  and the charged lepton mass matrix are real and diagonal. In this basis the Dirac mass matrix  $m_D$  contains all information about  $CP$  violation. We shall discuss now three possible parameterizations of  $m_D$ .

#### 1. Bi-unitary parametrization

With  $N$  light and  $N'$  heavy neutrinos, denoting this particle content as the “ $N \times N'$  see-saw model”, one can write the complex  $N \times N'$  Dirac mass matrix as

$$m_D = U_L^\dagger m_D^{\text{diag}} U_R, \quad (11)$$

where  $U_L$  ( $U_R$ ) is a unitary  $N \times N$  ( $N' \times N'$ ) matrix and  $m_D^{\text{diag}}$  is a real matrix with non-zero elements only at its  $ii$  entries. Thus,  $m_D^{\text{diag}}$  contains  $\min(N, N')$  real parameters we shall denote by  $m_{Di}$ . Any unitary  $N \times N$  matrix can be written as

$$U = e^{i\Phi} P \tilde{U} Q, \quad (12)$$

where  $P \equiv \text{diag}(1, e^{i\phi_1}, e^{i\phi_2}, \dots)$  and  $Q \equiv \text{diag}(1, e^{i\alpha}, e^{i\beta}, \dots)$  are diagonal phase matrices with  $(N-1)$  phases, and  $\tilde{U}$  is a unitary “CKM-like” matrix containing  $\frac{1}{2}(N-2)(N-1)$  phases and  $\frac{1}{2}N(N-1)$  angles. In total  $U$  contains  $\frac{1}{2}N(N+1)$  phases. Parameterizing  $U_L^\dagger$  and  $U_R$  in the same way, we get for the Dirac mass matrix

$$m_D = e^{i(\Phi_R + \Phi_L)} P_L \tilde{U}_L Q_L m_D^{\text{diag}} P_R \tilde{U}_R Q_R, \quad (13)$$

where the index  $L(R)$  indicates that the angles and phases in the respective matrices also carry this index. The common phase  $(\Phi_R + \Phi_L)$  and the  $(N-1)$  phases in  $P_L$  can be absorbed by a redefinition of the charged lepton fields. From the original  $(N+N'-2)$  phases in the matrices  $Q_L$  and  $P_R$ , only  $(N+N'-2) - (\max(N, N') - 1)$  appear in the product  $Q_L m_D^{\text{diag}} P_R$ . Thus, we can write

$$m_D = \tilde{U}_L W m_D^{\text{diag}} \tilde{U}_R Q_R, \quad (14)$$

where  $W$  is a diagonal matrix containing  $N'-1$  ( $N-1$ ) phases if  $N > N'$  ( $N \leq N'$ ). The number of physical  $CP$  violating phases is therefore

$$\begin{aligned} & \frac{1}{2}N(N+1) + \frac{1}{2}N'(N'+1) - 2 - (N-1) - (\max(N, N') - 1) \\ &= \frac{1}{2}N(N-1) + \frac{1}{2}N'(N'+1) - \max(N, N') \\ &= \frac{1}{2}N(N-3) + \frac{1}{2}N'(N'-3) + N' + \min(N, N') \end{aligned} \quad (15)$$

Adding the number of the real parameters  $m_{Di}$ ,  $\min(N, N')$ , of the masses  $M_i$ ,  $N'$ , and of the angles in  $\tilde{U}_L$  ( $\tilde{U}_R$ ),  $\frac{1}{2}N(N-1)$  ( $\frac{1}{2}N'(N'-1)$ ), we obtain the total number of parameters in the  $N \times N'$  see-saw model. The result is  $N(N-1) + N'(N'+1) - \max(N, N') + \min(N, N')$ . For  $N = N'$  this reduces to  $2N^2$  parameters,  $N(N-1)$  phases and  $N(N+1)$  real one. In general, the difference between the number of real parameters and phases is  $N + N'$ .

Comparing the number of low and high energy parameters, we see that integrating out the heavy Majorana neutrinos leave us short of a total of  $N'(N'+1) - N - \max(N, N') + \min(N, N')$  parameters, of which  $\frac{1}{2}N'(N'+1) - N + \min(N, N')$  are real and  $\frac{1}{2}N'(N'+1) - \max(N, N')$  are phases. For  $N = N'$  half of the parameters “get lost”,  $\frac{1}{2}N(N-1)$  phases and  $\frac{1}{2}N(N-1)$  real ones. The counting of the independent physical parameters in  $m_D$  through the independent parameters in  $U_{L,R}$  made above, is valid for  $|N - N'| = 0, 1$ . For arbitrary  $|N - N'| > 1$ , the total number of independent physical parameters in  $m_D$  is obviously  $N(2N'-1)$ , of which  $NN'$  are moduli and  $N(N'-1)$  are phases. Their number is smaller than the number our counting through  $U_{L,R}$  gives because when  $|N - N'| > 1$  there are more unphysical parameters in  $U_{L,R}$ .

The usual  $3 \times 3$  see-saw model has therefore 18 parameters, which are composed of 12 real parameters and 6 phases. Integrating out the heavy Majorana neutrinos leaves us with the observable mass matrix  $m_\nu$ , which contains 9 observable parameters — 6 real and 3 phases. Hence, half of the parameters of the model get “lost” at low energy. However, in approaches to reconstruct the high energy physics from low energy data, one usually assumes the existence of relations between mass matrices (e.g.,  $m_D = m_{\text{up}}$ ), which reduces the number of unknown parameters. If in addition specific textures in the matrices are implemented, the situation improves further.

It is worth noting that “reduced” models allow for a simpler connection between the high and low energy physics phenomena. Consider the minimal  $3 \times 2$  see-saw model [12, 13], which contains only 2 heavy Majorana neutrinos. This model has altogether 11 parameters, 8 real and 3  $CP$  violating phases. Comparing with the 9 observable parameters in  $m_\nu$ , we see that only 2 real parameters are “missing” at low energy. Interestingly, this makes the model superior to the even more reduced “too minimal”  $2 \times 2$  see-saw model with 2 heavy and 2 light Majorana neutrinos<sup>2</sup>. The latter has 8 parameters, two of which are phases, and this has to be compared with the 4 parameters (including 1 phase) a  $2 \times 2$  light Majorana neutrino mass matrix effectively contains.

We shall use in the analysis that follows the bi-unitary parametrization. Before proceeding further, however, we will describe briefly two alternative parameterizations.

## 2. Triangular parametrization

This parametrization relies on the property that any complex matrix can be written as a product of a unitary and a lower triangular matrix [14]:

$$m_D = v U Y . \quad (16)$$

The triangular matrix  $Y$  has zeros as entries above the diagonal axis and contains 3 phases in the three off-diagonal entries. The unitary matrix  $U$  can be parametrized in analogy to the PMNS matrix, i.e., in the form of a CKM-like matrix with one phase times a diagonal matrix containing the other two phases. An analysis of leptogenesis using this parametrization was performed in Ref. [15].

## 3. Orthogonal parametrization

The following parametrization shows clearly that without any assumptions there is no connection between the low and high energy parameters governing respectively neutrino mixing and leptogenesis. One can write the Dirac matrix as [43]

$$m_D = i U_{\text{PMNS}} \sqrt{m_\nu^{\text{diag}}} R \sqrt{M_R} , \quad (17)$$

where  $R$  is a complex orthogonal matrix. It contains 3 real parameters and 3 phases, which, together with the 3  $M_i$  and the 9 parameters from  $m_\nu$ , sums up again to a total of 18 parameters. In Eq. (17),  $m_D$  seems to contain all 18 parameters. However, the product  $\sqrt{m_\nu^{\text{diag}}} R \sqrt{M_R}$  includes only 9 independent parameters.

## 3.2 Leptogenesis, $m_\nu$ and LFV Charged Lepton Decays

The decay asymmetry  $\varepsilon_1$  depends on the three mass eigenvalues  $M_i$  and on the hermitian matrix  $m_D^\dagger m_D$ . Due to its hermiticity, the latter will have a reduced number of parameters with respect to  $m_D$ : for every row (or column) we get rid of one complex number, or equivalently of one real parameter and one phase. Indeed,

$$m_D^\dagger m_D = \begin{cases} U_R^\dagger (m_D^{\text{diag}})^2 U_R , & \text{bi-unitary;} \\ v^2 Y^\dagger Y , & \text{triangular;} \\ \sqrt{M_R} R^\dagger m_\nu^{\text{diag}} R \sqrt{M_R} , & \text{orthogonal,} \end{cases} \quad (18)$$

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<sup>2</sup>This model can be motivated by a vanishing mixing angle  $\theta_3$ . The  $\nu_e$  oscillates then into the flavor state  $(\nu_\mu + \nu_\tau)/\sqrt{2}$  with an amplitude  $\sin^2 2\theta_\odot$ .



depends on  $15 - 6 = 9$  parameters, which for the bi-unitary parametrization are the 3 angles and 3 phases in  $U_R$  and the 3 mass eigenvalues  $m_{Di}$ ; for the triangular parametrization, these are the 6 real entries and 3 phases in  $Y$ . In the orthogonal parametrization we have the 9 parameters from the combination  $\sqrt{m_\nu^{\text{diag}}} R \sqrt{M_R}$ . Of course, there are three leptogenesis phases in each case.

The form of  $m_D^\dagger m_D$  in the orthogonal parametrization shows that the PMNS matrix  $U_{\text{PMNS}}$  does not enter into the expression for  $\varepsilon_1$  and that there is, in general, no connection between the low energy observables in  $U_{\text{PMNS}}$  and leptogenesis. In particular, there is no connection between the low and high energy  $CP$  violation. Figure 1 shows a sketch of the situation: in order to connect low energy lepton charge non-conservation and  $CP$  violation effects (making an assumption about the currently unknown values of the phases and the lowest neutrino mass eigenstate) with the baryon asymmetry of the Universe  $Y_B$ , there must exist a connection between the light left-handed neutrinos  $\nu_L$  and the heavy right-handed Majorana neutrinos  $N_R$ . This typically involves some kind of see-saw mechanism within the framework of a GUT theory. The  $N_R$  produce  $Y_B$  via the leptogenesis mechanism. Without the crucial GUT or see-saw link there is no connection.

Inserting the form of  $m_D$  from Eqs. (11) and (16) in Eq. (4) shows <sup>3</sup> that all six phases in  $m_D$  contribute to the phases in  $U_{\text{PMNS}}$ :

$$m_\nu = \begin{cases} -U_L^\dagger m_D^{\text{diag}} U_R M_R^{-1} U_R^T m_D^{\text{diag}} U_L^* , & \text{bi-unitary;} \\ -v^2 U Y M_R^{-1} Y^T U^T , & \text{triangular.} \end{cases} \quad (19)$$

One may expect that a connection between the high and low energy  $CP$  violation would exist if three of the phases in  $m_D$  are negligible, so that the numbers of the  $CP$  violating phases at high and low energy coincide. In order to have successful leptogenesis, the negligible phases should not be those appearing in  $Y$  or  $U_R$ . Thus, one can assume that, e.g.,  $U$  in Eq. (16) is real, as has been done in [15]. We shall pursue a different approach and consider the bi-unitary parametrization keeping all the phases.

In supersymmetric versions of the see-saw mechanism, the rates of LFV charged lepton decays, such as  $\mu \rightarrow e + \gamma$ , and the  $T$  violating asymmetries in, e.g.,  $\mu \rightarrow 3e$  decay, depend approximately <sup>4</sup> on [42, 43, 44]

$$m_D m_D^\dagger = \begin{cases} U_L^\dagger (m_D^{\text{diag}})^2 U_L , & \text{bi-unitary;} \\ v^2 U Y Y^\dagger U^\dagger , & \text{triangular;} \\ U_{\text{PMNS}} \sqrt{m_\nu^{\text{diag}}} R M_R R^\dagger \sqrt{m_\nu^{\text{diag}}} U_{\text{PMNS}}^\dagger , & \text{orthogonal.} \end{cases} \quad (20)$$

In principle, taking also a possible data on the electric dipole moments of charged leptons into account, there are enough observables for a full reconstruction of the see-saw model. The bi-unitary parametrization seems to be especially convenient for this purpose.

<sup>3</sup>Note that inserting Eq. (17) in (4) yields an identity.

<sup>4</sup>The actual dependence is a bit more evolved, see Section 2.2.

## 4 The bi-Unitary Parametrization for $m_D$

As we saw, the  $3 \times 3$  see-saw model has 6 phases as possible sources of leptonic  $CP$  violation. The 3 low energy  $CP$  violating phases depend, in general, on all 6 see-saw phases. The unitary matrices  $U_L$  and  $U_R$  have a simple interpretation:  $U_L$  diagonalizes  $m_D m_D^\dagger$ , thus being responsible for lepton flavor violation, and  $U_R$  diagonalizes  $m_D^\dagger m_D$ , therefore being responsible for the total lepton number non-conservation. Since the latter is a necessary ingredient of leptogenesis and leads also to  $(\beta\beta)_{0\nu}$ -decay, one might assume naively that there will be a certain connection between the  $(\beta\beta)_{0\nu}$ -decay rate and  $Y_B$ . We shall see that within certain plausible assumptions this is indeed the case.

The first step is to determine the form of  $U_L$  and  $U_R$ . Since in any GUT theory  $m_D$  is related to the known charged fermion masses, we can assume a hierarchical structure of  $m_D$ , i.e.,  $m_{D3} \gg m_{D2} \gg m_{D1}$ , such that

$$m_D \simeq \text{diag}(m_{D1}, m_{D2}, m_{D3}) + \mathcal{O}(\text{few } \%), \quad (21)$$

where the second term indicates that there are corrections not exceeding 10 % on both the diagonal and off-diagonal entries of  $m_D$ . It is helpful to parametrize  $U_L^\dagger$  and  $U_R$  in analogy to  $U_{\text{PMNS}}$  in Eq. (2), with the angles  $\theta_i$  replaced by  $\theta_{L(R)i}$ . In what regards the  $CP$  violating phases, we shall denote the phases in  $\tilde{U}_R$  and  $\tilde{U}_L^\dagger$  with  $\delta_R$  and  $\delta_L$  respectively, the two ‘‘Majorana phases’’ in  $Q_R$  by  $\alpha_R$ ,  $\beta_R + \delta_R$  and the two phases in  $W$  by  $\alpha_W$  and  $\beta_W + \delta_L$ . In order to obtain a hierarchical  $m_D$  we shall assume that  $s_{L(R)1} \sim 10^{-1} > s_{L(R)2} \sim 10^{-2} > s_{L(R)3}$  with  $s_{L(R)3} \lesssim 10^{-3}$ , as well as that  $m_{D3} \gg m_{D2} \gg m_{D1}$ . These assumptions are inspired by observed mixing in the quark sector and by the known hierarchies between the masses of the up-type quarks, between the masses of the down-type quarks and between the charged lepton masses. In the numerical estimates we give in what follows we will always use the values  $m_{D1} \sim 100$  MeV,  $m_{D2} \sim 1$  GeV and  $m_{D3} \sim 100$  GeV, which to a certain degree are suggested by the up-quark mass values.

Once  $U_L^\dagger$  ( $U_R$ ) is chosen to contain hierarchical mixing angles, the requirement that  $m_D$  takes the form (21) implies hierarchical mixing angles also for  $U_R$  ( $U_L^\dagger$ ). The structure of the Dirac matrix is then found to be

$$m_D \simeq \begin{pmatrix} m_{D1} - \tilde{m}_{D2} s_{1L} s_{1R} & e^{i\alpha_R} \tilde{m}_{D2} s_{1L} & e^{i(\beta_R + \delta_R - \delta_L)} \tilde{m}_{D3} s_{3L} \\ -\tilde{m}_{D2} s_{1R} & e^{i\alpha_R} \tilde{m}_{D2} & e^{i(\beta_R + \delta_R)} \tilde{m}_{D3} s_{2L} \\ \tilde{m}_{D3} (s_{1R} s_{2R} - e^{i\delta_R} s_{3R}) & -e^{i\alpha_R} \tilde{m}_{D3} s_{2R} & e^{i(\beta_R + \delta_R)} \tilde{m}_{D3} \end{pmatrix}, \quad (22)$$

where  $\tilde{m}_{D2} \equiv m_{D2} e^{i\alpha_W}$ ,  $\tilde{m}_{D3} \equiv m_{D3} e^{i(\beta_W + \delta_L)}$  and we have set all the  $\cos \theta_{L(R)i}$  to one. Products of the  $\sin \theta_{L(R)i}$  which are smaller than  $10^{-3}$  are neglected. We have neglected also terms of order  $\mathcal{O}(m_{D3} s_{2L} s_{2R})$  with respect to  $m_{D2}$ . Not surprisingly, the off-diagonal entries of  $m_D$  are suppressed with respect to the diagonal terms by the small quantities  $s_{iL(R)}$ ,  $i = 1, 2, 3$ . A hierarchical structure of  $m_D$  is most naturally described using the bi-unitary parametrization.

Rather than tuning the parameters involved in the bi-unitary parametrization to reproduce precisely the neutrino mass squared differences and mixing angles determined from the

available neutrino oscillation data, we shall focus only on the dependence of the quantities of interest on the parameters in  $m_D$  and  $M_R$ : we are primarily interested to find out under which circumstances a connection between the neutrino mixing related phenomena and leptogenesis is possible. We made nevertheless extensive consistency checks regarding the form of  $m_\nu$  as obtained with the matrix  $m_D$  in Eq. (22) and with  $M_R$ . Using the form of  $m_D$  given in Eq. (22) and assuming a hierarchy for  $M_i$ ,  $M_1 \ll M_2 \ll M_3$ , from Eq. (4) one can obtain a theoretically predicted expression for the low energy neutrino mass matrix,  $m_\nu^{\text{th}}$ . The form of  $m_\nu^{\text{th}}$  should be confronted with the neutrino mass matrix which is obtained from Eq. (1),  $m_\nu^{\text{exp}}$ . For the angles  $\theta_1 \equiv \theta_\odot$  and  $\theta_2 \equiv \theta_{\text{atm}}$ , we use the best fit values found in the analyzes [5, 6] of the solar and atmospheric neutrino data, respectively:  $\tan^2 \theta_\odot = 0.46$  and  $\sin^2 2\theta_{\text{atm}} = 1$ . The angle  $\theta_3$  is varied within its allowed  $3\sigma$  range,  $\sin^2 \theta_3 < 0.05$ , while the three  $CP$  violating phases  $\delta$ ,  $\alpha$  and  $\beta$  are treated as free parameters. For the normal hierarchical neutrino mass spectrum one has:  $m_1 \ll m_2 \simeq \sqrt{\Delta m_\odot^2} \ll m_3 \simeq \sqrt{\Delta m_{\text{atm}}^2}$ , where  $\Delta m_\odot^2 > 0$  and  $\Delta m_{\text{atm}}^2 > 0$  drive the solar and atmospheric neutrino oscillations. In the numerical estimates which follow we use  $\Delta m_\odot^2 = 7.3 \times 10^{-5} \text{ eV}^2$  [5] and  $\Delta m_{\text{atm}}^2 = 2.7 \times 10^{-3} \text{ eV}^2$  [6].

The expression for  $m_\nu^{\text{th}}$  and the analysis of interest simplify considerably if the hierarchy between the masses  $M_1$  and  $M_2$  is relatively mild:

$$M_2 \simeq 10 M_1 . \quad (23)$$

It is not difficult to convince oneself that if Eq. (23) holds and if in addition — in agreement with the hierarchy requirement in  $m_D$  — the relation

$$m_{D3} s_{2R(L)} \simeq 10^{-2} m_{D3} \simeq m_{D2} \quad (24)$$

holds, the form of  $m_\nu^{\text{th}}$  leads to  $\sin^2 2\theta_2 \equiv \sin^2 2\theta_{\text{atm}} \simeq 1$  without any further fine-tuning as long as the terms  $\propto 1/M_3$  in Eq. (26) give a sub-leading contribution to  $m_\nu^{\text{th}}$ . This is ensured provided the inequality

$$M_3 \gg s_{2R}^{-2} M_2 \sim 10^4 M_2 , \quad (25)$$

implying a strong hierarchy between  $M_2$  and  $M_3$ , holds. The existence of a mild hierarchy between  $M_1$  and  $M_2$ , Eq. (23), and of a strong hierarchy between  $M_2$  and  $M_3$ , Eq. (25) is, as we will see, also compatible with the requirement of effective leptogenesis.

Under the conditions (23) — (25) the matrix  $m_\nu^{\text{th}}$  takes a relatively simple form:

$$\begin{aligned}
(m_\nu^{\text{th}})_{ee} &\simeq \frac{(m_{D1} - s_{1L}s_{1R}m_{D2}e^{i\alpha_W})^2}{M_1} + \frac{s_{1L}^2 m_{D2}^2 e^{2i(\alpha_W + \alpha_R)}}{M_2} \\
(m_\nu^{\text{th}})_{e\mu} &\simeq -\frac{(m_{D1} - s_{1L}s_{1R}m_{D2}e^{i\alpha_W})s_{1R}m_{D2}e^{i\alpha_W}}{M_1} + \frac{m_{D2}^2 s_{1L} e^{2i(\alpha_W + \alpha_R)}}{M_2} \\
(m_\nu^{\text{th}})_{e\tau} &\simeq \frac{(m_{D1} - s_{1L}s_{1R}m_{D2}e^{i\alpha_W})(s_{1L}s_{2R} - s_{3R}e^{i\delta_R})m_{D3}^2 e^{2i(\beta_W + \delta_L)}}{M_1} - \\
&\quad \frac{m_{D2}m_{D3}s_{1L}s_{2R}e^{i(\alpha_W + \beta_W + \delta_L + 2\alpha_R)}}{M_2} \\
(m_\nu^{\text{th}})_{\mu\mu} &\simeq -\frac{m_{D2}^2}{M_2} e^{2i(\alpha_R + \alpha_W)} , \\
(m_\nu^{\text{th}})_{\mu\tau} &\simeq \frac{m_{D2}m_{D3}}{M_2} s_{2R} e^{i(2\alpha_R + \alpha_W + \beta_W + \delta_L)} , \\
(m_\nu^{\text{th}})_{\tau\tau} &\simeq -\frac{m_{D3}^2}{M_2} s_{2R}^2 e^{2i(\alpha_R + \beta_W + \delta_L)} ,
\end{aligned} \tag{26}$$

where we have not written explicitly the sub-leading terms  $\propto 1/M_1$  and  $\propto 1/M_3$ . Further analysis shows that for the generic values of the parameters  $m_{Di}$  and  $s_{jL(R)}$  we use, the matrix  $m_\nu^{\text{th}}$  thus obtained leads to a value of  $\theta_\odot \equiv \theta_1$  compatible with the observations for, e.g.,  $M_2 \sim 10^{10}$  GeV. The value of  $\theta_\odot$  is particularly sensitive to the specific value of  $m_{D1}$  chosen: a very good agreement with the best fit value  $\tan^2 \theta_\odot \cong (0.42 - 0.46)$  determined in the analyses of the solar neutrino data is obtained for  $m_{D1} \cong (70 - 90)$  MeV.

In what regards the angle  $\theta_3$ , for  $m_{D1} \sim 100$  MeV and  $s_{1L} \simeq (0.10 - 0.25)$  and the standard values of the other parameters we use, one generically has:  $\sin \theta_3 \sim s_{1L}/\sqrt{2}$ . Consequently,  $\sin^2 \theta_3$  is relatively large: typically one has  $\sin^2 \theta_3 \gtrsim 0.005$ . However, it is also possible to fine-tune the values of the parameters involved in the analysis to get  $\sin^2 \theta_3 < 0.005$  and this latter possibility cannot be ruled out.

The above analysis shows that the assumptions we made about the magnitude of the angles in  $U_L$  and  $U_R$  and of the values of  $m_{Di}$  are very well in agreement with the existing data. We find, in particular, that the angle  $\theta_3$  can be relatively large ( $\sin^2 \theta_3 \gtrsim 0.005 - 0.010$ ), which has important implications for the searches for  $CP$  violation in neutrino oscillations. In fact, we shall see that under the assumptions made, the  $CP$  violating observables in neutrino oscillations can be sizable.

## 4.1 Leptogenesis in the bi-Unitary Parametrization

In the context of leptogenesis the lepton asymmetry depends on the parameter  $m_D^\dagger m_D$ , as discussed in Section 2. Using Eq. (18) we find for  $(m_D^\dagger m_D)_{11}$  to leading order:

$$\begin{aligned}
(m_D^\dagger m_D)_{11} &\simeq m_{D2}^2 s_{1R}^2 + m_{D3}^2 (s_{1R}^2 s_{2R}^2 - 2 \cos \delta_R s_{1R} s_{2R} s_{3R} + s_{3R}^2) \\
&\simeq (m_{D2}^2 + m_{D3}^2 s_{2R}^2) s_{1R}^2 .
\end{aligned} \tag{27}$$

The decay asymmetry  $\varepsilon_1$  receives contribution from

$$\begin{aligned} & \text{Im} \left\{ (m_D^\dagger m_D)_{12}^2 \right\} \simeq \\ & 2 (\cos \alpha_R s_{1R} (m_{D2}^2 + m_{D3}^2 s_{2R}^2) - m_{D3}^2 \cos(\alpha_R - \delta_R) s_{2R} s_{3R}) \\ & (\sin \alpha_R s_{1R} (m_{D2}^2 + m_{D3}^2 s_{2R}^2) - m_{D3}^2 \sin(\alpha_R - \delta_R) s_{2R} s_{3R}) \\ & \simeq (m_{D2}^2 + m_{D3}^2 s_{2R}^2)^2 s_{1R}^2 \sin 2\alpha_R, \end{aligned} \quad (28)$$

as well as from

$$\begin{aligned} & \text{Im} \left\{ (m_D^\dagger m_D)_{13}^2 \right\} \simeq \\ & 2 m_{D3}^4 (\cos(\beta_R + \delta_R) s_{1R} s_{2R} - \cos \beta_R s_{3R}) (\sin(\beta_R + \delta_R) s_{1R} s_{2R} - \sin \beta_R s_{3R}) \\ & \simeq m_{D3}^4 s_{1R}^2 s_{2R}^2 \sin 2(\beta_R + \delta_R), \end{aligned} \quad (29)$$

where  $s_{3R}$  was neglected in the last expression. Assuming a hierarchical mass spectrum of the heavy Majorana neutrinos one finds for the decay asymmetry

$$\varepsilon_1 \simeq -\frac{3}{2} \frac{1}{8\pi v^2} \left( (m_{D2}^2 + m_{D3}^2 s_{2R}^2) \sin 2\alpha_R \frac{M_1}{M_2} + \frac{m_{D3}^4 s_{2R}^2}{m_{D2}^2 + m_{D3}^2 s_{2R}^2} \sin 2(\beta_R + \delta_R) \frac{M_1}{M_3} \right). \quad (30)$$

Therefore, we can identify  $\alpha_R$  and  $(\beta_R + \delta_R)$  as the leptogenesis phases. Note that, as it should,  $\varepsilon_1 = 0$  for  $\alpha_R, \beta_R, \delta_R = 0, \pi/2, \pi$ . Taking  $m_{D2} \simeq 10^{-2} m_{D3}$  and  $s_{2R} \simeq 10^{-2}$ , we can estimate the decay asymmetry as <sup>5</sup>

$$\varepsilon_1 \sim -10^{-9} \left( \frac{m_{D3}}{\text{GeV}} \right)^2 \left( \sin 2\alpha_R \frac{M_1}{M_2} + 10^3 \sin 2(\beta_R + \delta_R) \frac{M_1}{M_3} \right). \quad (31)$$

As discussed earlier, the heavy Majorana neutrino masses  $M_i$  are also expected to possess a hierarchy. Under the condition (25) of a strong hierarchy between  $M_2$  and  $M_3$ , the second term in the last equation is negligible, and only the phase  $\alpha_R$  is relevant for leptogenesis. If further one has  $m_{D3} \sim 10^2$  GeV, as is suggested by up-quark mass values, it is possible to have  $\varepsilon_1 \sim 10^{-6} - 10^{-7}$  if, e.g.,  $M_1/M_2 \sim 0.1$ , which is compatible with the requirement of a mild hierarchy between  $M_1$  and  $M_2$ , Eq. (23) which in the previous Section was shown to be also compatible with the low energy neutrino phenomenology.

Identifying  $m_D$  with the down-quark or charged lepton mass matrix, i.e.,  $m_{D3} \sim 1$  GeV, leads to a rather small value of  $\varepsilon_1$  for  $M_1 \ll M_2 \ll M_3$ . In this case the lepton asymmetry can only be amplified to the requisite value by the resonance mechanism [33] which is operative for quasi-degenerate heavy Majorana neutrinos. Thus, leptogenesis with hierarchical neutrino Majorana and Dirac masses prefers up-quark type masses for the latter, a fact first noticed in [17].

The “effective mass”  $\tilde{m}_1$ , that represents a sensitive parameter for the Boltzmann equations, is given by

$$\tilde{m}_1 = \frac{(m_D^\dagger m_D)_{11}}{M_1} \sim \left( \frac{m_{D3}}{\text{GeV}} \right)^2 \left( \frac{10^9 \text{GeV}}{M_1} \right) 10^{-5} \text{eV}. \quad (32)$$

---

<sup>5</sup>Note that the form of this equation resembles Eq. (17) in the second paper in Ref. [8]. A type II see-saw mechanism was used in that work.

For  $m_{D3} \sim 10^2$  GeV and  $M_1$  of  $10^9$  to  $10^{10}$  GeV, acceptable values of  $10^{-1}$  to  $10^{-3}$  eV for  $\tilde{m}_1$  are obtained. This leads to values of  $\kappa \sim 10^{-1} - 10^{-3}$ . Assuming that the sine of the phases in Eq. (30) is not small, a value of  $Y_B \sim 10^{-10}$  is therefore “naturally” obtained for, e.g.,  $m_{D3} \sim 10^2$  GeV,  $M_1 \sim 10^9$  GeV, and  $M_2 \sim 10^{10}$  GeV  $\ll M_3 \sim 10^{15}$  GeV. The indicated values of  $M_1$ ,  $M_2$  and  $M_3$  are also in very good agreement with the constraints Eqs. (23) and (25).

## 4.2 LFV Charged Lepton Decays

According to Eq. (9), the branching ratios of the LFV charged lepton decays,  $BR(\ell_i \rightarrow \ell_j + \gamma)$ , depend on  $(m_D L m_D^\dagger)$ . We have calculated this matrix with the usual approximations for the masses and angles and found the leading terms to be

$$\begin{aligned} (m_D L m_D^\dagger)_{21} &\simeq m_{D2}^2 s_{1L} L_2, \\ (m_D L m_D^\dagger)_{31} &\simeq m_{D3}^2 (s_{1L} s_{2L} - s_{3L} e^{i\delta_L}) L_3 + m_{D3} m_{D2} s_{1L} s_{2R} (L_2 - L_3). \\ (m_D L m_D^\dagger)_{32} &\simeq m_{D3}^2 s_{2L} L_3. \end{aligned} \quad (33)$$

Barring accidental cancellations in the expression for  $(m_D L m_D^\dagger)_{31}$ , we obtain the following general relation between the branching ratios of interest <sup>6</sup>:

$$BR(\tau \rightarrow \mu + \gamma) \gg BR(\tau \rightarrow e + \gamma) \gg BR(\mu \rightarrow e + \gamma). \quad (34)$$

If we use the generic values of the relevant parameters we work with, we find

$$BR(\tau \rightarrow \mu + \gamma) \sim 10^2 BR(\tau \rightarrow e + \gamma) \sim 10^5 BR(\mu \rightarrow e + \gamma). \quad (35)$$

The results do not differ significantly from the results obtained by using a common mass scale  $M$  and taking the log-terms out of the matrix, i.e., for  $(m_D m_D^\dagger) \log M_X/M$  instead of  $m_D L m_D^\dagger$ . Note that though  $m_D$  contains  $U_L$  and  $U_R$ , only the angles and phases of  $U_L$  appear in  $BR(\ell_i \rightarrow \ell_j + \gamma)$ , which confirms that Eq. (20) represents a rather good approximation for estimating  $BR(\ell_i \rightarrow \ell_j + \gamma)$ .

## 5 Leptogenesis, $(\beta\beta)_{0\nu}$ -decay and Low Energy Leptonic $CP$ Violation

We shall investigate next whether there exists a relation between the high energy  $CP$  violation leading to the decay asymmetry Eq. (30) and low energy observables. We will consider the effective Majorana mass in  $(\beta\beta)_{0\nu}$ -decay and the  $CP$  violating asymmetry in neutrino oscillations.

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<sup>6</sup>Obviously, the relation we obtain does not hold if, e.g., the accidental cancellation  $(s_{1L} s_{2L} - s_{3L} e^{i\delta_L}) = 0$  takes place. The latter requires, however, a fine-tuning of the values of 4 parameters.

## 5.1 The Effective Majorana Mass in $(\beta\beta)_{0\nu}$ -Decay

The observation of  $(\beta\beta)_{0\nu}$ -decay would provide direct evidence for the non-conservation of the total lepton charge, which is a necessary ingredient for the leptogenesis mechanism. In  $(\beta\beta)_{0\nu}$ -decay one measures the absolute value of the  $ee$  element of  $m_\nu$ . Given the form of  $m_D$  in Eq. (22), it is not difficult to find this element:

$$\langle m \rangle \simeq \frac{(m_{D1} - s_{1L}s_{1R}m_{D2}e^{i\alpha_W})^2}{M_1} + \frac{m_{D2}^2 e^{2i(\alpha_W+\alpha_R)}}{M_2} + \frac{m_{D3}^2 s_{3L}^2 e^{2i(\beta_R+\beta_W+\delta_R)}}{M_3}. \quad (36)$$

The term of order  $m_{D3}^2$  contains, in particular, the parameters  $\alpha_W$  and  $\beta_W$ , which do not influence the baryon asymmetry. The condition that this term does not contribute significantly to  $|\langle m \rangle|$  is

$$\frac{m_{D3}^2}{M_3} s_{3L}^2 \ll \begin{cases} \frac{m_{D2}^2}{M_1} s_{1L}^2 s_{1R}^2 \\ \frac{m_{D2}^2}{M_2} s_{1L}^2 \end{cases} \Rightarrow M_3 \gg \begin{cases} M_1 \left( \frac{m_{D3}}{m_{D2}} \right)^2 \frac{s_{3L}^2}{s_{1L}^2 s_{1R}^2} \sim 10^2 M_1 \\ M_2 \left( \frac{m_{D3}}{m_{D2}} \right)^2 \frac{s_{3L}^2}{s_{1L}^2} \sim M_2 \end{cases}, \quad (37)$$

which is compatible with Eqs. (24) and (26) and the hierarchy between  $M_3$  and  $M_{1,2}$ , required for generating a sufficient baryon asymmetry (see the discussion after Eq. (31)). The first two terms in (36) can contribute comparably to  $|\langle m \rangle|$  unless  $M_2 s_{1R}^2 \gg M_1$  or  $M_2 s_{1R}^2 \ll M_1$ . From the two main possibilities for  $m_D$ , the choice of the up-quark type matrix,  $m_D \sim m_{\text{up}}$ , favored by leptogenesis in the case of hierarchical  $M_i$  and  $m_D$ , leads to  $|\langle m \rangle|$  which is larger by a factor of  $(m_c/m_{s,\mu})^2 \sim 10^2$  then in the case of  $m_D \sim m_{\text{down}}$ ,  $m_c$  ( $m_s$ ) being the charm (strange) quark mass. If  $m_{D2} \sim 1$  GeV and  $s_{1L(R)}^2 \sim (10^{-2} - 10^{-1})$  (see Eq. (25)), one finds for  $M_1 \sim 10^9$  GeV and  $M_2 \sim 10^{10}$  GeV that  $|\langle m \rangle| \sim (0.001 - 0.01)$  eV<sup>7</sup>, which may be within the reach of the 10t version of the GENIUS experiment [45].

If condition (37) holds, only the leptogenesis parameter  $\alpha_R$  appears in the effective Majorana mass  $|\langle m \rangle|$ . This condition corresponds to a rather strong hierarchy between the masses of the  $N_i$  and thus to a decoupling of the heaviest Majorana neutrino  $N_3$ . In this case, one sees from Eq. (30) that only the term proportional to  $\sin 2\alpha_R$  contributes to  $Y_B$  and there is a direct correlation between the rate of  $(\beta\beta)_{0\nu}$ -decay and the baryon asymmetry of the Universe. For a mild hierarchy between the masses  $M_i$ , the unknown phases  $\alpha_W$  and  $\beta_W$  spoil any simple connection between  $|\langle m \rangle|$  and  $Y_B$ .

## 5.2 $CP$ Violation in Leptogenesis and in Neutrino Oscillations

Manifest  $CP$  violation can be probed in neutrino oscillation experiments (see, e.g., [24]). The corresponding  $CP$  violating observables depend on the rephasing invariant quantity [16]  $J_{CP}$  — the leptonic analog of the Jarlskog invariant. The following form [15] of  $J_{CP}$  is particularly suited for our analysis:

$$J_{CP} = -\frac{\text{Im}(h_{12} h_{23} h_{31})}{\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2}, \text{ where } h = m_\nu m_\nu^\dagger. \quad (38)$$

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<sup>7</sup>We would like to emphasize that the numerical values for the observables we give should be taken not too literally: a spread within one order of magnitude, as included in the prediction for  $|\langle m \rangle|$  we give, should be allowed for.

The invariant  $J_{CP}$  can be calculated using Eqs. (4) and (22). The resulting expression is rather long and which is the leading term depends on the degree of hierarchy of the heavy Majorana neutrino masses. In general, however, we find that  $J_{CP}$  vanishes for  $m_{D2} = 0$  and/or  $1/M_3 = 0$ . These approximations correspond effectively to a 2 flavour neutrino mixing case, and consequently to an absence of Dirac-like  $CP$  violation. In general, all six phases contribute to  $J_{CP}$ , as is suggested by Eq. (19).

As an example, let us consider the hierarchy  $M_1 \simeq 10^{-1} M_2 \simeq 10^{-4} M_3$ . Then there are four leading terms in  $J_{CP}$ , which are proportional to  $(M_1 M_2^4 M_3)^{-1}$ ,  $(M_1 M_3)^{-3}$ ,  $(M_1 M_2 M_3^4)^{-1}$  and  $(M_1 M_2 M_3)^{-2}$ , respectively. They read:

$$\begin{aligned} \text{Im}(h_{12} h_{23} h_{31}) \simeq & \frac{-2 m_{D2}^5 m_{D3}^7 s_{1L}^2 s_{1R}^2 s_{2L} s_{2R}^5}{M_1 M_2^4 M_3} \cos 2\alpha_R \sin(\alpha_W - (\beta_W + \delta_L) + 2\alpha_R - 2(\beta_R + \delta_R)) \\ & - \frac{m_{D2}^5 m_{D3}^7 s_{1L}^2 s_{1R}^5 s_{2L} s_{3R}}{M_1^3 M_3^3} \sin(\alpha_W - 2\beta_R - \beta_W - \delta_L - \delta_R) \\ & + \frac{2 m_{D2}^4 m_{D3}^8 s_{1L}^2 s_{1R}^2 s_{2L} s_{3L}}{M_1 M_2 M_3^4} \cos 2\alpha_R \sin \delta_L \\ & + \frac{m_{D2}^7 m_{D3}^5 s_{1L}^2 s_{1R}^4 s_{2L} s_{2R}}{M_1^2 M_2^2 M_3^2} (2 \sin(\alpha_W - \beta_W - \delta_L) + \sin(\alpha_W - 4\alpha_R - \beta_W - \delta_L)) \\ & + \frac{m_{D2}^5 m_{D3}^5 s_{1L}^2 s_{1R}^4 s_{2L} s_{2R}}{M_1^2 M_2^2 M_3^2} (m_{D2}^2 (2 \sin(\alpha_W - \beta_W - \delta_L) + \sin(\alpha_W - 4\alpha_R - \beta_W - \delta_L)) \\ & \quad + m_{D3}^2 s_{2R}^2 \sin(\alpha_W - \beta_W - \delta_L)) , \end{aligned} \tag{39}$$

which illustrates our general conclusions mentioned above. Choosing a different hierarchy of the heavy Majorana neutrino masses will lead to the presence of different terms with different combinations of the parameters, especially of the phases.

Of the 6 independent physical  $CP$  violating phases of the see-saw model, 5 phases are present in Eq. (39), namely  $\alpha_R$ ,  $\alpha_W - \beta_W - \delta_L$ ,  $\beta_R$ ,  $\delta_R$  and  $\delta_L$ . This can be understood as being due to the fact that in the approximations we use  $m_{D1}$  is neglected. Setting in addition  $s_{3L(R)}$  to zero should remove two more phases from the parameter space. Indeed, one finds from Eq. (39) that only three independent phases ( $\alpha_R$ ,  $\alpha_W - \beta_W - \delta_L$  and  $\beta_R + \delta_R$ ) enter into the expression for  $J_{CP}$  in this case. Order-of-magnitude-wise we can predict the magnitude of the  $CP$  violation to be

$$J_{CP} \sim 10^{-2} \left( \frac{10^9 \text{ GeV}}{M_1} \right) \left( \frac{10^{10} \text{ GeV}}{M_2} \right)^4 \left( \frac{10^{14} \text{ GeV}}{M_3} \right) \sin(\alpha_W - (\beta_W + \delta_L) + 2\alpha_R - 2(\beta_R + \delta_R)) . \tag{40}$$

One sees that within the approximation discussed in this paper there is no connection, in general, between leptogenesis in Eq. (31) and the low energy  $CP$  violation in neutrino oscillations. Successful leptogenesis without low energy  $CP$  violation in neutrino oscillations is, in principle, possible. This interesting case has recently been discussed in [11]. However, given the results of the present Subsection, fine-tuning between the values of the different phases has to take place in order to have leptogenesis and  $J_{CP} \approx 0$ . Even if there is no direct connection between the leptogenesis and the value of  $J_{CP}$ , a measurement of  $J_{CP}$  can shed light on the “non-leptogenesis” parameters. We note finally that the decoupling of leptogenesis from the low energy Dirac-phase has been noticed to take place in a number of



models as well, e.g. in [10].

We comment finally on an interesting special case of Eq. (39). Suppose that the phases “conspire” to fulfill the relations  $\alpha_W - \beta_W - \delta_L = \beta_R = \delta_R = \delta_L = 0$  or, for  $s_{3L(R)} = 0$ ,  $\alpha_W - \beta_W - \delta_L = \beta_R + \delta_R = 0$ . Then, only one phase contributes to the  $J_{CP}$  asymmetry and one finds from Eq. (39) that  $J_{CP} \propto \sin 4\alpha_R$ . The baryon asymmetry, on the other hand, is proportional to  $\sin 2\alpha_R$  as can be seen from Eqs. (7) and (30). This situation allows for a correlation between the relative sign of the baryon asymmetry of the Universe and the  $CP$  asymmetry in neutrino oscillations. The two quantities possess opposite signs if  $2\alpha_R$  lies between  $\pi/2$  and  $\pi$  and the same signs for values of  $2\alpha_R$  outside this range. The parameter space allowing for this correlation corresponds to the  $3 \times 2$  see-saw model with 2 texture zeros in the Dirac mass matrix discussed in Ref. [12].

## 6 Conclusions

Assuming only a hierarchical structure of the heavy Majorana neutrino masses  $M_{1,2,3}$ ,  $M_1 \ll M_2 \ll M_3$ , and of the Dirac mass matrix  $m_D$  of the see-saw mechanism (see Eq. (21)), and working in the bi-unitary parametrization of  $m_D$ , we find that in order to produce a sufficient amount of baryon asymmetry via the leptogenesis mechanism, the scale of  $m_D$  should be given by the up-quark masses. In this class of “hierarchical” see-saw models, the branching ratios of LFV charged lepton decays, whose dependence on  $m_D$  is introduced by RGE effects within the SUSY GUT version of the model, are predicted to fulfill the relations  $BR(\tau \rightarrow \mu + \gamma) \gg BR(\tau \rightarrow e + \gamma) \gg BR(\mu \rightarrow e + \gamma)$ : typically one has  $BR(\tau \rightarrow \mu + \gamma) \sim 10^2 BR(\tau \rightarrow e + \gamma) \sim 10^5 BR(\mu \rightarrow e + \gamma)$ . We find that the effective Majorana mass in  $(\beta\beta)_{0\nu}$ -decay depends on the  $CP$  violating phase controlling the leptogenesis if one of the heavy Majorana neutrinos is much heavier than the other two,  $M_3 \gg 10^4 M_2$ . A rather mild hierarchy between the masses of the lighter two heavy Majorana neutrinos,  $M_2 \sim 10 M_1$ , is required for successful leptogenesis. The hierarchical relations  $M_3 \gg 10^4 M_2$  and  $M_2 \sim 10 M_1$  with, e.g.,  $M_1 \sim 10^9$  GeV, are also compatible with the low-energy neutrino mixing phenomenology. The  $CP$  violation effects in neutrino oscillations can be observable. In general, there is no direct connection between the latter and the  $CP$  violation in leptogenesis. We find, however, that if the  $CP$  violating phases of the see-saw model “conspire” to satisfy certain relations, the baryon asymmetry of the Universe and the leptonic  $CP$  violation rephasing invariant  $J_{CP}$ , which determines the magnitude of the  $CP$  violation effects in neutrino oscillations, depend on the same  $CP$  violating phase and their signs are correlated.

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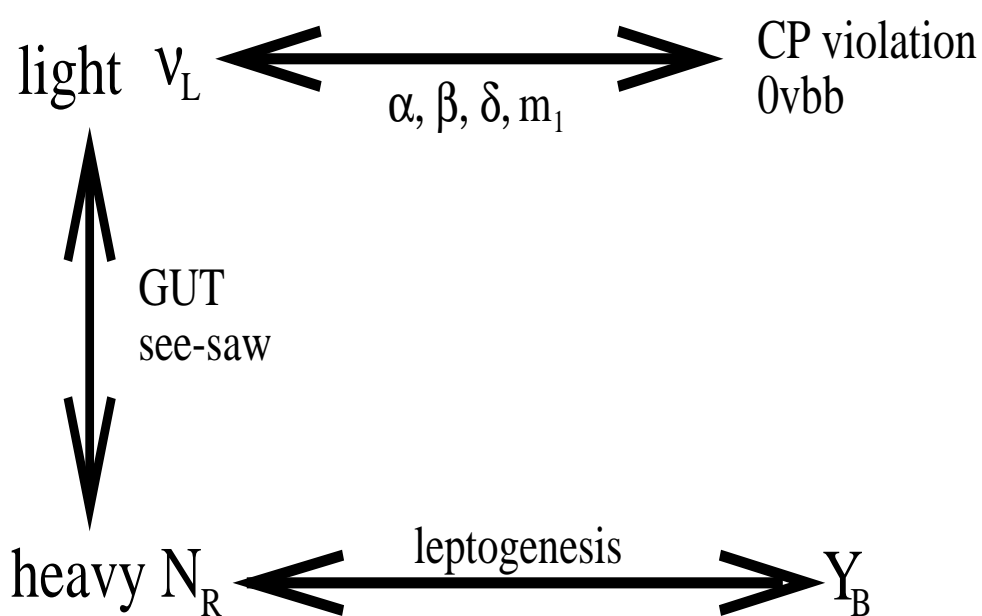


Figure 1: Connection between low energy lepton number and  $CP$  violation with the baryon asymmetry  $Y_B$  via the leptogenesis mechanism. Without the left vertical arrow there is none.